# Microeconomics Comps June 2008

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1.

2. Profit maximization.

(a) AKI's profit maximization problem is

$$\max_{f,l} p_k f^a l^b - p_f f$$
$$\text{s.t. } l \leq g f^a l^b \text{ and } f > 0$$

(b) We have

$$l \leq g f^a l^b \Rightarrow l^{1-b} \leq g f^a$$

Since profit is strictly increasing with $l$, $l = g^{\frac{1}{1-b}} f^{\frac{a}{1-b}}$. Therefore, the problem is equivalent to

$$\max_{f,l} p_k g^{\frac{b}{1-b}} f^{\frac{a}{1-b}} - p_f f$$
$$\text{s.t. } f > 0$$

If $a < 1 - b$, then the objective function is strictly concave and the supply correspondence is determined by first order condition.

(c) The objective function is strictly convex, will supply infinity.

3. General equilibrium with insurance market.

(a) A competitive A-D market is a price $p > 0$ and an allocation $x = (x_a, x_d)$, such that

- given $p$, $x$ solves individual maximization problem

$$\max \frac{1}{2}u_a(x_a) + \frac{1}{2}u_d(x_d)$$
$$\text{s.t. } x_a + p x_d = k$$

- market clearing

$$\frac{1}{2}x_a + \frac{1}{2}x_d = \frac{1}{2}k$$

A competitive A-D equilibrium is $p = 1$ and $x^*$ satisfying

$$x_a^* > 0 \quad x_d^* > 0 \quad x_a^* + x_d^* = k$$
$$u_a'(x_a^*) = u_d'(x_d^*)$$

(b) First consider the workers' part. If a worker does not participate in the insurance program, her expected payoff is

$$\frac{1}{2}u_d(0) + \frac{1}{2}u_a(k) = \frac{1}{2}u_a(k) \tag{1}$$

If a worker does participate and tell truth, her expected payoff is

$$\frac{1}{2}u_d(T-q) + \frac{1}{2}u_a(k-q) \tag{2}$$

If a worker participates and lies, her expected payoff is

$$\frac{1}{2}u_d(T-q) + \frac{1}{2}u_a(T-q) \tag{3}$$

Comparing (2) and (3) tells if $T > k$, a worker who participates in insurance will lie.

Second consider the insurance companies' part. Assume either (2) or (3) is greater than (1). Therefore it is beneficial for every worker to participate in the insurance program. Since the market is competitive, insurance company will provide the most beneficial contract to workers and make zero profit.

To sum up, a competitive equilibrium is a $(T, q)$, $T \geq q$ such that

- $x_a = \max\{T-q, k-q\}$ and $x_d = T - q$;
- $x = (x_a, x_d)$ solves
$$\max \frac{1}{2}u_d(x_d) + \frac{1}{2}u_a(x_a) \tag{4}$$
- (4)≥(1);
- Companies make non-negative profit.

From assumption given in question, we know that $x_a > x_d$. Hence $k > T$, which means a worker who is able to work at time 1 will not lie. Therefore, an equilibrium is

$$T - q = x_d^* \quad k - q = x_a^* \Rightarrow q = k - x_a^* \quad T = k - x_a^* + x_d^*$$

Also, $T = 2q$, so insurance company makes zero profit.

(c) If $x_d > x_a$, then workers who are able to work will lie in time 1 given her participation in the insurance program. Therefore, the payment of the company becomes $T$ instead of $\frac{1}{2}T$, which leads to negative profit. Hence $x^*$ is not implementable. An equilibrium is $T = q$ and autarky allocation.

4. Externality.

(a) A optimizes.

(b) Conditional on $k$, socially optimizes.

(c) Do a comparison between A's different offers.

(d) Depends.

(e) Can't be better off. Maybe worse off.